

# Ground-plane screening of Coulomb interactions in two-dimensional systems: How effectively can one two-dimensional system screen interactions in another

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(Received 21 April 2009; revised manuscript received 24 July 2009; published 5 October 2009)

The use of a nearby metallic ground-plane to limit the range of the Coulomb interactions between carriers is a useful approach in studying the physics of two-dimensional (2D) systems. This approach has been used to study Wigner crystallization of electrons on the surface of liquid helium, and most recently, the insulating and metallic states of semiconductor-based two-dimensional systems. In this paper, we perform calculations of the screening effect of one 2D system on another and show that a 2D system is at least as effective as a metal in screening Coulomb interactions. We also show that the recent observation of the reduced effect of the ground-plane when the 2D system is in the metallic regime is due to intralayer screening.

DOI: [10.1103/PhysRevB.80.155412](https://doi.org/10.1103/PhysRevB.80.155412)

PACS number(s): 71.30.+h, 71.10.-w, 71.45.Gm

## I. INTRODUCTION

In a two-dimensional electron system (2DES), strong Coulomb interactions between electrons can lead to exotic phenomena such as the Wigner crystal state,<sup>1-3</sup> the fractional quantum Hall effect,<sup>4,5</sup> and the anomalous 2D metallic state.<sup>6-8</sup> One route to studying the role played by Coulomb interactions is to limit their length scale using a metallic ground-plane located close to the 2DES.<sup>9,10</sup> This approach was first used in studies of the melting of the Wigner crystal state formed in electrons on a liquid He surface.<sup>11,12</sup> More recently, it has been used to study the role of Coulomb interactions in the insulating<sup>13</sup> and metallic<sup>14</sup> regimes of a 2D hole system (2DHS) formed in an AlGaAs/GaAs heterostructure.

Whereas the study of Coulomb interactions in the insulating regime<sup>13</sup> was achieved quite straightforwardly using a metal surface gate separated from the 2DHS by  $\sim 500$  nm [see Fig. 1(a)], the corresponding study in the metallic regime could not be achieved in this way. This is because the higher hole density  $p$  in the metallic regime requires that the distance  $d$  between the 2DHS and ground-plane be comparable to the carrier spacing [ $d \sim 2(\pi p)^{-1/2} \sim 50$  nm] to achieve effective screening, and at the same time that the 2DHS be deep enough in the heterostructure ( $> 100$  nm) to achieve a mobility sufficient to observe the metallic behavior. To overcome this challenge, a double quantum well heterostructure was used [see Fig. 1(b)] such that the 2DHS formed in the upper quantum well (screening layer) served as the ground-plane for the lower quantum well (transport layer), enabling the measurement of a  $\sim 340$  nm deep, high mobility 2DHS separated by only 50 nm from a ground-plane.<sup>14</sup>

In considering experiments on screening in double quantum well systems, a natural question to ask is whether a 2D system is as effective as a metal gate when used as a ground-plane to screen Coulomb interactions between carriers in a nearby 2D system. This is important given that the screening charge in a 2D system is restricted to two dimensions and the density of states is several orders of magnitude smaller than in a metal film. In this paper, we perform calculations of the

screening effect of a ground-plane on a 2D system for two cases: the first where the ground-plane is a metal and the second where it is a 2D system. We begin using the Thomas-Fermi approximation in the absence of intralayer screening in the transport layer to show that a 2D system is at least as effective as a metal gate as a ground-plane for the experiment in Ref. 14. We also compare the experiments in the insulating<sup>13</sup> and metallic<sup>14</sup> regimes of a 2D hole system in the Thomas-Fermi approximation, to explain why the ground-plane has less effect in the metallic regime compared to the insulating regime. Finally, since the experiment by Ho *et al.* was performed at  $r_s > 1$ , where the Thomas-Fermi approximation begins to break down, we extend our model to account for exchange and finite thickness effects to see how

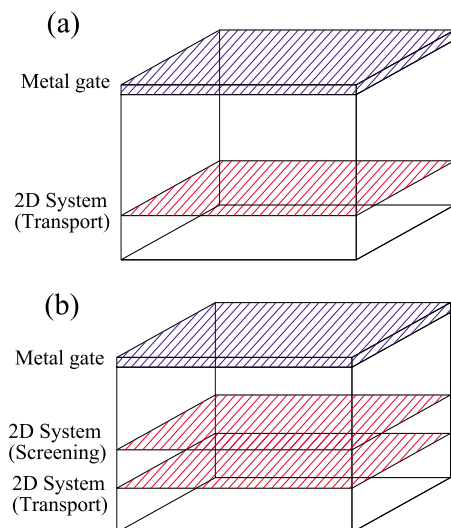


FIG. 1. (Color online) Schematics of the ground-plane screening experiments recently performed by (a) Huang *et al.* (Ref. 13) and (b) Ho *et al.* (Ref. 14). In (b), there are two possible ground-plane configurations. In the first, the gate is grounded and the 2D system acts as the ground-plane. In the second, the gate is biased to deplete the upper 2D system and the gate then acts as the ground-plane instead. This allows the distance between the transport layer and the ground-plane to be varied *in situ*—For more details, see Ref. 14.

these affect the conclusions from the Thomas-Fermi model.

The paper is structured as follows. In Sec. II we derive the dielectric functions for screening of a 2D hole system by a metal gate and another nearby 2D hole system. In Sec. III, we compare the various dielectric functions numerically and discuss their implications for the ground-plane screening experiments of 2D systems in the insulating<sup>13</sup> and metallic<sup>14</sup> regimes. Conclusions will be presented in Sec. IV. For readers unfamiliar with the intricacies of screening in 2D systems, we give a brief introduction to the screening theory for a single 2D system in Appendix A to aid them in understanding the theory developed in Sec. II. In Appendix B, we compare our model accounting for exchange and finite thickness effects to related works on many-body physics in double quantum well structures. In Appendix C we show how our analysis of ground-plane screening can be related to previous work on bilayer 2D systems that have studied the compressibility. In contrast to experiments on ground-plane screening of the interactions within a 2D system,<sup>13,14</sup> previous experiments on bilayer 2D systems have examined the compressibility of a 2D system by studying penetration of an electric field applied perpendicular to the 2D plane.<sup>15</sup> We show that although these two screening configurations have quite distinct geometries, our model is consistent with widely used analysis<sup>15</sup> describing the penetration of a perpendicular electric field through a 2D system.

In the calculations that follow, we use linear screening theory and the static dielectric function approximation (i.e.,  $\omega \rightarrow 0$ ). Unless otherwise specified, we assume for convenience that the 2D systems contain holes (electron results can be obtained with appropriate corrections for charge and mass) to facilitate direct connection with recent experimental results in AlGaAs/GaAs heterostructures.<sup>13,14</sup> We also assume that tunneling between the two quantum wells is negligible and ignore any Coulomb drag effects (i.e., interlayer exchange and correlations).

## II. SCREENING OF ONE 2D LAYER BY ANOTHER

We now begin considering the screening effect of a nearby ground-plane on a 2D system (transport layer) for two different configurations. In the first, the ground-plane (i.e., screening layer) is a metal surface gate [see Fig. 2(a)] and in the second, the ground-plane is another 2D system [see Fig. 2(b)]. In both cases the transport and screening layers are separated by a distance  $d$ .

If we consider some positive external test charge  $\rho_1^{\text{ext}}$  added to the transport layer, this leads to induced charge in both the transport layer  $\rho_1^{\text{ind}}$  (as in Appendix A) and in the screening layer  $\rho_2^{\text{ind}}$ . Note however that no external charge is added to the screening layer, so  $\rho_2^{\text{ext}}=0$  and  $\rho_2=\rho_2^{\text{ind}}$ , whereas  $\rho_1=\rho_1^{\text{ext}}+\rho_1^{\text{ind}}$ .

How we deal with the induced charge in the screening layer differs in the two cases. In both cases, we consider the transport layer and a second layer of charge a distance  $D$  above it; each layer having a potential and charge density of  $\phi_1(q), \rho_1(q)$  and  $\phi_2(q), \rho_2(q)$ , respectively. For a metal surface gate, we can use the standard image charge approach,<sup>16</sup> which involves considering the induced charge in the screen-

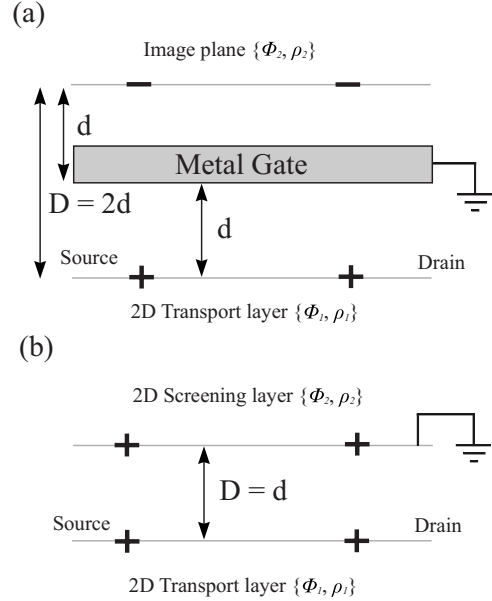


FIG. 2. Schematics showing the two systems considered in this paper. The transport layer is screened by (a) a metal surface gate and (b) a second 2D system. In both cases the screening layer is separated by a distance  $d$  from the transport layer, and the transport (1) and screening (2) layers have independent potentials  $\phi$  and charge densities  $\rho$ .

ing layer as a 2D layer of negative “image” charges located a distance  $D=2d$  away from the transport layer. This results in  $\rho_2^{\text{ind}}=-\rho_1$  for a metal gate. The image charge approach assumes that the ground-plane is a perfect metal. This assumption is relatively well satisfied by a typical metal surface gate (Au gate  $\sim 150$  nm thick) but not by a 2D system. Thus when a 2D system is used as the screening layer, we cannot assume an added induced negative image charge as we can for the metal. Instead, we account for screening by a 2D system by directly calculating its induced charge, which is located in the screening layer (i.e., at a distance  $D=d$ ). This results in  $\rho_2^{\text{ind}}=\chi^0\phi_2$  for a 2D screening layer, where  $\chi^0$  is the polarizability, which describes how much  $\rho^{\text{ind}}$  is produced in response to the addition of  $\rho^{\text{ext}}$ . Note that we are only considering the case where the both 2D layers are of the same type of charge (e.g., a bilayer 2DHS).

Charge in one layer leads to a potential in the other via the interlayer Coulomb interaction:<sup>17</sup>

$$U(q) = \mathcal{F} \left[ \frac{1}{4\pi\epsilon\sqrt{r^2 + D^2}} \right] = e^{-qD}V(q) \quad (1)$$

where  $V(q)=\frac{1}{2\epsilon q}$  is the intralayer Coulomb interaction,  $D$  is the distance between the two layers, and  $\mathcal{F}$  is the Fourier transform. If the screening layer is a metal  $D=2d$ , and  $D=d$  if it is a 2D system. The resulting potential in the transport layer then becomes:

$$\phi_1(\mathbf{q}) = V(q)\rho_1 + U(q)\rho_2^{\text{ind}}(\mathbf{q}). \quad (2)$$

We discuss how to obtain  $\rho_2^{\text{ind}}(\mathbf{q})$  in Sec. II A. The effectiveness of the screening is obtained from the modified dielectric function  $\epsilon(q, d)$ , which we define as the inverse of the ratio

of the screened potential to the unscreened potential for the transport layer:

$$\epsilon(q, d) = \left( \frac{\phi_1}{\phi_1^{\text{ext}}} \right)^{-1}. \quad (3)$$

The dielectric function for the transport layer can be obtained for three possible configurations: no screening layer (i.e., just a single 2D system), a 2D screening layer and a metal screening layer, which we denote as  $\epsilon_{\text{single}}(q)$ ,  $\epsilon_{2\text{D}}(q, d)$ , and  $\epsilon_{\text{metal}}(q, d)$ , respectively.  $\epsilon_{\text{single}}(q)$  is the conventional dielectric function for a single 2D system and can be recovered by taking the limit  $d \rightarrow \infty$  for either  $\epsilon_{2\text{D}}(q, d)$  or  $\epsilon_{\text{metal}}(q, d)$ , as we show in Sec. II C. If one wanted to take transport theory for a single layer and consider the effects of ground-plane screening, it is only necessary to replace  $\epsilon_{\text{single}}(q)$  with  $\epsilon_{2\text{D}}(q, d)$  or  $\epsilon_{\text{metal}}(q, d)$  to take into account the screening effect of a 2D or metal ground-plane, respectively.

We will now obtain the various  $\epsilon$  using an approach involving the random phase approximation<sup>18</sup> (RPA) and Thomas-Fermi (TF) approximation.<sup>19</sup> At first we will ignore (Sec. II A) and later include (Sec. II B) intralayer screening in the transport layer in the calculations. Finally, in Sec. II C, we will extend the model for a 2D screening layer to account for its behavior at lower densities, to confirm that our conclusions from the simpler calculations are robust.

#### A. No intralayer screening in the transport layer

We begin by considering the case where there is no intralayer screening in the transport layer. This is useful because it allows a straightforward comparison of the effectiveness of the 2DHS as a ground-plane, without the obscuring effect of intralayer screening. To do this calculation, we set  $\rho_1^{\text{ind}}=0$ , such that  $\rho_1=\rho_1^{\text{ext}}$ . In other words, there is only external charge in the transport layer and only induced charge in the screening layer.

Considering the metal gate first, we have  $\rho_2^{\text{ind}}(\mathbf{q})=-\rho_1(\mathbf{q})=-\rho_1^{\text{ext}}(\mathbf{q})$  from the method of images. If we combine the two results above for  $\rho_1$  and  $\rho_2^{\text{ind}}$  with Eq. (2), we obtain

$$\phi_1(\mathbf{q}) = [V(q) - U(q)]\rho_1^{\text{ext}}(\mathbf{q}). \quad (4)$$

After using Eq. (1) to eliminate  $U(q)$ , Eq. (3) then gives the dielectric function for the metal gate:

$$\frac{1}{\epsilon_{\text{metal,ns}}(q, d)} = 1 - e^{-2qd} \quad (5)$$

where the additional subscript ns denotes that intralayer screening has been ignored.

For a 2D screening layer, the dielectric function is obtained self-consistently through the RPA as follows. The induced charge is related to the screening layer potential by

$$\phi_2(\mathbf{q}) = U(q)\rho_1 + V(q)\rho_2^{\text{ind}}(\mathbf{q}) \quad (6)$$

$$\rho_2^{\text{ind}}(\mathbf{q}) = \chi_2^0(q)\phi_2(\mathbf{q}) \quad (7)$$

where  $\chi_2^0$  is the polarizability of the screening layer, normally given by the 2D Lindhard function.<sup>20</sup> When this is combined with Eq. (6), knowing that  $\rho_1=\rho_1^{\text{ext}}$ , we obtain

$$\rho_2^{\text{ind}} = \frac{\chi_2^0(q)V(q)}{1 - \chi_2^0(q)V(q)} e^{-qd} \rho_1^{\text{ext}}. \quad (8)$$

This result is substituted into Eq. (2), and using Eqs. (1) and (3) gives:

$$\frac{1}{\epsilon_{2\text{D,ns}}(q, d)} = 1 + \frac{\chi_2^0(q)V(q)}{1 - \chi_2^0(q)V(q)} e^{-2qd}. \quad (9)$$

To simplify this expression, we use the Thomas-Fermi approximation  $\chi_2^0(q)=-e^2 \frac{dn}{d\mu}$ , where  $\frac{dn}{d\mu}$  is the thermodynamic density of states<sup>21</sup> of the 2D system, to give

$$\frac{1}{\epsilon_{2\text{D,ns}}(q, d)} = 1 - \frac{q^{TF}}{q + q^{TF}} e^{-2qd} \quad (10)$$

where the Thomas-Fermi wavevector  $q^{TF} = \frac{m^* e^2}{2\pi\epsilon_0\epsilon_r\hbar^2}$ . Note that if we take the 2D screening layer to the metallic limit, in other words, we give it an infinite density of states, which corresponds to  $q^{TF} \rightarrow \infty$ , then Eq. (10) reduces to Eq. (5), as one would expect.

#### B. With intralayer screening in the transport layer

We now consider the case where there is intralayer screening (i.e., finite polarizability and induced charge) in the transport layer. To approach this problem, we again place an external charge density  $\rho_1^{\text{ext}}$  in the transport layer, but now we have induced charge in both the transport  $\rho_1^{\text{ind}}$  and screening  $\rho_2^{\text{ind}}$  layers. Additionally, we label the polarization  $\chi_i^0(q)$  and Thomas-Fermi wavenumber  $q_i^{TF}$  where  $i=1$  or 2 corresponding to the transport and screening layers, respectively. The derivation proceeds as before, but with the addition of the induced charge density  $\rho_1^{\text{ind}}(\mathbf{q})=\chi_1^0(q)\phi_1(\mathbf{q})$  in the transport layer. The results obtained are

$$\frac{1}{\epsilon_{\text{metal,s}}(q, d)} = \frac{1 - e^{-2qd}}{1 - V(q)\chi_1^0(q)(1 - e^{-2qd})} = \frac{1 - e^{-2qd}}{1 + \frac{q_1^{TF}}{q}(1 - e^{-2qd})} \quad (11)$$

for the metal gate, where the added subscript s denotes that intralayer screening has been included, and

$$\begin{aligned} \frac{1}{\epsilon_{2\text{D,s}}(q, d)} &= \frac{1 - V\chi_2^0[1 - e^{-2qd}]}{[1 - V\chi_1^0][1 - V\chi_2^0] - V^2\chi_1^0\chi_2^0 e^{-2qd}} \\ &= \frac{1 + \frac{q_2^{TF}}{q}[1 - e^{-2qd}]}{\left(1 + \frac{q_1^{TF}}{q}\right)\left(1 + \frac{q_2^{TF}}{q}\right) - \frac{q_1^{TF}q_2^{TF}}{q^2} e^{-2qd}} \quad (12) \end{aligned}$$

when the screening layer is a 2D system.

We can check the consistency of these equations with those in Sec. II A in three ways. Firstly, by setting  $q_1^{TF}=0$ , which corresponds to no screening or induced charge in the transport layer, Eqs. (11) and (12) reduce to Eqs. (5) and (10), respectively. Secondly, if we set  $q_2^{TF}=0$  instead, which corresponds to no screening or induced charge in the

screening layer, then  $\epsilon_{2D,s}^{-1}$  in Eq. (12) reduces to  $\epsilon_{\text{single}}^{-1}$ , which is given in the Thomas-Fermi approximation by:<sup>21</sup>

$$\frac{1}{\epsilon_{\text{single}}(q)} = \left(1 + \frac{q_1^{TF}}{q}\right)^{-1}. \quad (13)$$

Finally, if we set  $q_2^{TF} \rightarrow \infty$  to take the 2D screening layer to the metallic limit, then  $\epsilon_{2D,s}^{-1}$  in Eq. (12) reduces to  $\epsilon_{\text{metal,s}}^{-1}$  in Eq. (11).

### C. More accurate calculations for 2D systems at lower densities

Following our relatively simple treatment of ground-plane screening above, it is now interesting to ask how the results of our calculations change if we extend our model to account for two phenomena ignored in our Thomas-Fermi model: exchange effects at low densities and the finite thickness of the screening and transport layers.

The Thomas-Fermi approximation works well when the interaction parameter  $r_s = (a_B^* \sqrt{\pi p})^{-1} \lesssim 1$ , where  $a_B^* = 4\pi\epsilon\hbar^2/m^*e^2$  is the effective Bohr radius. However, it is not as accurate for 2D systems at lower densities, such as those used in our experiment,<sup>14</sup> where the interaction parameters for the screening and transport layers were  $r_s \sim 10$  and  $10.2 < r_s < 14.3$ , respectively. At such low densities, it is essential to include the effects of exchange, and a better approximation involves the addition of the local field correction,<sup>22</sup> which we will use in this section.

One might question if linear screening theory is valid at the large  $r_s$  values considered here, or whether it is necessary to include nonlinear screening effects. Recent compressibility measurements of 2D systems across the apparent metal-insulator transition<sup>23</sup> have shown that the divergence of the inverse compressibility at low densities is consistent with nonlinear screening theories,<sup>24,25</sup> which take into account a percolation transition in the 2D system. This consistency suggested that at sufficiently low densities, nonlinear theories were required to adequately describe screening. However, although we are interested in fairly high  $r_s$ , our analysis relates to measurements performed on the metallic side of the apparent metal-insulator transition, where the effect of inhomogeneities is minimal. Indeed, Ref. 25 shows a comparison of the nonlinear screening theories<sup>24,25</sup> with a standard uniform screening theory. These theories all yield the same results in our density range of interest, confirming that

it is safe to treat the 2D system as homogenous and use linear screening in our calculations.

Returning to the inclusion of the local field correction to our calculations; when considering the case of two 2D layers, we will use the single layer local-field factor  $G(\mathbf{q})$  to account for intralayer exchange effects, and for simplicity, ignore any corresponding interlayer effects. In this work we will use the Hubbard approximation for  $G(\mathbf{q})$  (see Eq. (A6) in Appendix A). This leads to

$$\chi_i(q) = \frac{\chi_i^0(q)}{1 - V(q)\chi_i^0(q)[1 - G_i(q)]} \quad (14)$$

where  $G_i(q)$  and  $\chi_i^0$  are the local-field factor and 2D Lindhard function<sup>20</sup> for layer  $i=(1,2)$ , respectively. The calculations proceed as before in Sec. II B, except that where we consider intralayer screening in the transport layer, we have

$$\rho_1^{\text{ind}}(\mathbf{q}) = \chi_1(\mathbf{q})[\phi_1^{\text{ext}}(\mathbf{q}) + U(\mathbf{q})\rho_2^{\text{ind}}(\mathbf{q})]. \quad (15)$$

and where the ground plane is a 2D layer, we have

$$\rho_2^{\text{ind}}(\mathbf{q}) = \chi_2(\mathbf{q})[U(\mathbf{q})\rho_1^{\text{ext}}(\mathbf{q}) + U(\mathbf{q})\rho_1^{\text{ind}}(\mathbf{q})] \quad (16)$$

It is also important to account for the finite thickness of the screening and transport layers, which are confined to 20-nm-wide quantum wells in Ref. 14. To do this, we introduce a form factor  $F(q)$  that modifies the bare Coulomb interaction such that  $V(q) \rightarrow V(q)F(q)$ .<sup>26</sup> The form factor is defined as  $F(q) = \iint |\psi(z)|^2 |\psi(z')|^2 e^{-q|z-z'|} dz dz'$ , where  $\psi(z)$  is the wavefunction of an electron/hole in the direction perpendicular to the plane of the quantum well.<sup>27</sup> Assuming an infinite-square potential for the quantum well, we obtain:<sup>28,29</sup>

$$F(q) = \frac{1}{4\pi^2 + a^2 q^2} \left[ 3aq + \frac{8\pi^2}{aq} - \frac{32\pi^4(1 - e^{-aq})}{a^2 q^2(4\pi^2 + a^2 q^2)} \right] \quad (17)$$

where  $a$  is the width of the well. We thus obtain the dielectric functions  $\epsilon(q,d)$  as defined in Eq. (3), where  $\phi_1(\mathbf{q})$  remains as defined in Eq. (2), giving

$$\frac{1}{\epsilon_{2D,ns,xf}(q,d)} = 1 + \frac{Y_2 e^{-2qd}}{1 - Y_2[1 - G_2(q)]} \quad (18)$$

$$\frac{1}{\epsilon_{\text{metal,s,xf}}(q,d)} = \frac{(1 - e^{-2qd})[1 + Y_1 G_1(q)]}{1 - Y_1(1 - G_1(q) - e^{-2qd})} \quad (19)$$

$$\frac{1}{\epsilon_{2D,s,xf}(q,d)} = \frac{1 + Y_1 G_1(q) - Y_2[1 - G_2(q) - e^{-2qd}] + Y_1 Y_2 [G_1(q)G_2(q) - G_1(q)(1 - e^{-2qd})]}{\{1 - Y_1[1 - G_1(q)]\}\{1 - Y_2[1 - G_2(q)]\} - Y_1 Y_2 e^{-2qd}} \quad (20)$$

where  $Y_i = \chi_i^0(q)F(q)V(q)$  and the additional subscript  $xf$  indicates the inclusion of exchange and finite thickness effects. As a consistency check, if we take the 2D screening layer to

the metallic limit, by using  $G_1(q) = G_2(q) = 0$ ,  $Y_2 = -q_2^{TF}/q$ , and the limit  $q_2^{TF} \rightarrow \infty$ , and return to zero thickness  $F(q) = 1$ , then  $\epsilon_{2D,ns,xf}$  in Eq. (18) reduces to  $\epsilon_{\text{metal,ns}}$  in Eq. (5), and



$\epsilon_{2D,s,xf}$  in Eq. (20) reduces to  $\epsilon_{\text{metal},s}$  in Eq. (11). If we separate the two layers by taking  $d \rightarrow \infty$  then both  $\epsilon_{2D,s,xf}$  in Eq. (20) and  $\epsilon_{\text{metal},s,xf}$  in Eq. (19) reduces to  $\epsilon_{\text{single}}$  described by Eqs. (A3) and (A5) in Appendix A. We compare this work with related studies by Zheng and MacDonald<sup>30</sup> in Appendix B.

### III. RESULTS AND DISCUSSION

In this section, we will use the various dielectric functions derived in Sec. II to answer an important physical question regarding recent experiments on screening long-range Coulomb interactions in 2D systems: does a 2D system screen as effectively as a metal when used as a ground-plane?

We will answer this question in three stages. First we will consider the simplest possible case where there is no intralayer screening in the transport layer and the Thomas-Fermi approximation holds. Our results at this stage are directly applicable to ground-layer screening studies of dilute 2D systems, such as those investigating Wigner crystallization on liquid helium<sup>11,12</sup> and the 2D insulating state in an AlGaAs/GaAs heterostructure.<sup>13</sup> They may also be relevant to recent studies of the metal-insulator transition in Si metal-oxide-semiconductor field-effect transistors,<sup>31</sup> where the gate is likely to produce significant ground-plane screening in the nearby 2DES located  $<40$  nm away, for example. Second, we will then look at what happens when intralayer screening is introduced to the transport layer. This will allow us to understand why the ground-plane has such a significant effect on the insulating state in the experiment by Huang *et al.*<sup>13</sup> and such little effect on the metallic state in the experiment by Ho *et al.*<sup>14</sup> Finally, since the experiments in Refs. 13 and 14 were performed at  $r_s \gg 1$ , we will investigate how our results change if we extend beyond the Thomas-Fermi approximation and begin to account for finite thickness and exchange effects.

#### A. Thomas-Fermi approximation in the absence of intralayer screening

To get an understanding of the basic physics of our ground-plane screening model, we will begin by ignoring any effects of intralayer screening in the transport layer and use the Thomas-Fermi approximation to obtain the polarizability  $\chi(\mathbf{q})$ . There are two important parameters in our equations: the layer separation  $d$  and the wave-number  $q$  and to simplify our analysis we will make these parameters dimensionless by using  $q/q^{TF}$  and  $d^{TF} = d \times q^{TF}$  hereafter. The Thomas-Fermi wave-number  $q^{TF}$  contains all of the relevant materials parameters involved in the experiment. In Ref. 14, where measurements were performed using holes in GaAs,  $\epsilon_r = 12.8$ , and  $m^* = 0.38m_e$ , giving  $q^{TF} = 1.12 \times 10^9 \text{ m}^{-1}$  [i.e.,  $(q^{TF})^{-1} = 0.89 \text{ nm}$ ]. The corresponding values for electrons with  $m^* = 0.067m_e$  are  $q^{TF} = 1.97 \times 10^8 \text{ m}^{-1}$  and  $(q^{TF})^{-1} = 5.06 \text{ nm}$ . Table I presents the  $d$  values corresponding to the four  $d^{TF}$  values that we will discuss in Secs. III A and III B. The first two values correspond to  $d = 50 \text{ nm}$  for holes and electrons, the remaining two allow us to demonstrate

TABLE I.  $d$  values for holes and electrons corresponding to the four  $d^{TF}$  values considered in Secs. III A and III B.

$d^{TF}$	56.1	9.89	3	1
$d_{\text{holes}} \text{ (nm)}$	50	8.80	2.67	0.89
$d_{\text{electrons}} \text{ (nm)}$	283.87	50	15.18	5.06

what happens as the screening layer gets much closer to the transport layer in both cases.

To facilitate a comparative analysis of the effectiveness of the screening, in Fig. 3(a) we plot the inverse dielectric function  $\epsilon^{-1}$  vs  $q/q^{TF}$ , with both a metal (solid blue/dotted green lines) and a 2D system (dashed red lines) as the screening layer, for the four different  $d^{TF}$  values listed in Table I. Note that for the metal screening layer case ( $\epsilon_{\text{metal,ns}}(q,d)$  in Eq. (5)), we have explicitly parameterized  $q$  and  $d$  into  $q/q^{TF}$  and  $d^{TF}$ , in order to plot the metal and 2D screening layer cases on the same axes. The metal data for  $d^{TF} = 56.1$  is presented as dotted green line as it serves as reference data for later figures. Note that  $\epsilon^{-1} = 1$  corresponds to no screening and  $\epsilon^{-1} = 0$  corresponds to complete screening of a test charge placed in the transport layer. Considering the large  $d^{TF}$  limit first,  $\epsilon^{-1}$  only deviates from 1 at small  $q/q^{TF}$ , and heads towards  $\epsilon^{-1} = 0$  as  $q/q^{TF} \rightarrow 0$ . In other words, screening is

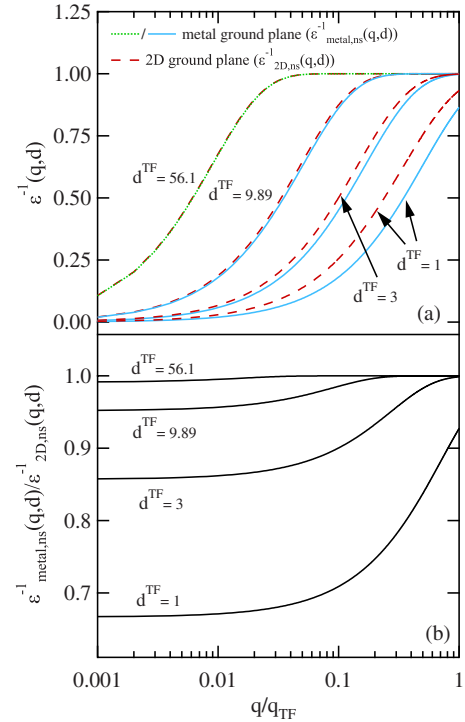


FIG. 3. (Color online) (a) The inverse dielectric function  $\epsilon^{-1}(q,d)$  vs  $q/q_{TF}$  with no intralayer screening in the transport layer. Data is presented for metal (solid blue and dotted green lines) and 2D (dashed red lines) screening layers for the four  $d_{TF}$  values presented in Table I. The metal gate data for  $d_{TF} = 56.1$  appears as a dotted green line as it serves as reference data for later figures. (b) The relative effectiveness of the ground-plane screening due to a 2D screening layer compared to a metal screening layer, as quantified by the ratio  $\epsilon_{\text{metal,ns}}^{-1}/\epsilon_{2D,ns}^{-1}$  vs  $q/q_{TF}$  for the four  $d_{TF}$  values.

only effective at large distances from a test charge added to the transport layer. This makes physical sense if one considers the electrostatics of ground-plane screening. The ground-plane acts by intercepting the field lines of the test charge such that they are no longer felt in other parts of the transport layer. This is only effective at distances from the test charge that are much greater than the ground-plane separation  $d$  and thus the ground-plane acts to limit the range of the Coulomb interaction in the transport layer, as pointed out by Peeters.<sup>9</sup> With this in mind, it is thus clear why the point of deviation from  $\epsilon^{-1}=1$  shifts to higher values of  $q/q_{TF}$  as  $d^{TF}$  is reduced. Indeed, all four lines pass through a common  $\epsilon^{-1}$  value when  $q/q_{TF} = \frac{1}{d^{TF}}$  reflecting this electrostatic aspect of ground-plane screening.

Turning to the central question of the effectiveness of a 2D layer as a ground-plane, in Fig. 3(a) it is clear from the increasing discrepancy between the solid and dashed lines that the 2D system becomes less effective than a metal as  $d^{TF}$  is reduced. To quantify this, in Fig. 3(b) we plot the ratio of the two dielectric constants  $\epsilon_{\text{metal,ns}}^{-1}/\epsilon_{\text{2D,ns}}^{-1}$ , with a ratio of 1 indicating equivalent screening and  $<1$  indicating that a 2D system is less effective than a metal. For large separations, for example  $d^{TF}=56.1$ , which corresponds directly to the experiment by Ho *et al.*, a 2D system screens as effectively as the metal gate to within 1%. However if the screening layer is brought very close to the transport layer  $d^{TF} \sim 1$  (i.e., the screening layer is only a Thomas-Fermi screening length away from the transport layer) then the effectiveness of the 2D system as a ground-plane is reduced to  $\sim 66\%$  of that of a metal layer at an equivalent distance. It is important to note that correlations between the two layers can be significant for such small separations and hence this increasing discrepancy should be considered as a qualitative result only. Furthermore, as we will see in Sec. III C, exchange actually acts to enhance the effectiveness of the 2D system as a ground-plane, making the Thomas-Fermi result above a significant underestimate of the true ground-plane screening of a 2D system in the low-density limit.

### B. Thomas-Fermi approximation with intralayer screening in the transport layer

We now add intralayer screening in the transport layer to our Thomas-Fermi model and begin by asking: what is the magnitude of this intralayer screening contribution, independent of any ground-plane screening effects? In Fig. 4(a), we plot the inverse dielectric function  $\epsilon_{\text{single}}^{-1}$  (dash-dotted black line) for a 2D system with intralayer screening and *no* nearby ground-plane. For comparison, we also show the data from Fig. 3(a) for a metal ground-plane with  $d_{TF}=56.1$  (green dotted line) and the expectation with no screening,  $\epsilon^{-1}=1$  for all  $q/q_{TF}$  (grey dashed horizontal line) in Fig. 4(a). It is clear that the addition of intralayer screening has a very significant impact on the dielectric function, more so than the addition of a ground-plane. Indeed, returning to an electrostatic picture and ignoring exchange and correlation effects,  $\epsilon_{\text{single}}^{-1}$  should assume the  $d^{TF} \rightarrow 0$  limit of  $\epsilon_{\text{2D,ns}}^{-1}$ , the Thomas-Fermi model in the absence of intralayer screening.

We now reintroduce the ground-plane and in Fig. 4(a), we plot the combined screening contributions for metal

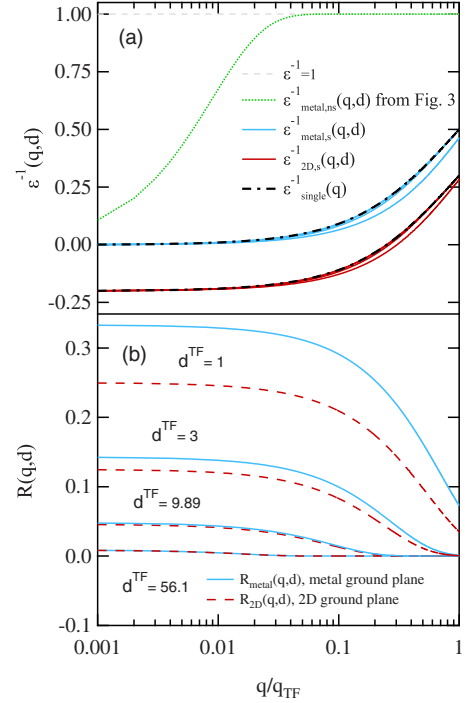


FIG. 4. (Color online) Effect of a screening layer on a 2D system with intralayer screening. (a) Firstly, in order to show the relative effects of intralayer and ground-plane screening, we plot the dielectric functions  $\epsilon^{-1}=1$  corresponding to no intralayer or ground-plane screening (grey dashed horizontal line),  $\epsilon_{\text{metal,ns}}^{-1}$  with metal screening layer at  $d_{TF}=56.1$  and no intralayer screening [dotted green line, data from Fig. 3(a)], and  $\epsilon_{\text{single}}^{-1}$  with intralayer screening but no ground-plane (dash-dotted black line). We then consider the effect of the metal screening layer when the intralayer screening is included by plotting  $\epsilon_{\text{metal,s}}^{-1}(q,d)$  (solid blue lines) for the four values of  $d^{TF}$  shown in Table I. Moving through the traces from upper left to lower right corresponds to decreasing  $d^{TF}$ . A similar set of curves are shown for the case of a 2D screening layer [ $\epsilon_{\text{2D,s}}^{-1}(q,d)$ , solid red lines] which has been offset vertically by  $-0.2$  for clarity, along with a duplicate of  $\epsilon_{\text{single}}^{-1}$  (dash-dotted black line). Since the dielectric functions almost lie on top of each other when intralayer screening is present, in (b) we plot  $R$ , the relative enhancement of  $\epsilon^{-1}$  due to the ground plane. Calculations for the four different values of  $d_{TF}$  in Table I are shown for metal (solid blue lines) and 2D (dashed red lines) ground-planes.

( $\epsilon_{\text{metal,s}}^{-1}(q,d)$ , solid blue lines) and 2D ( $\epsilon_{\text{2D,s}}^{-1}(q,d)$ , solid red lines) screening layers. These are shown for the four different values of  $d^{TF}$  in Table I. The values for the 2D system are offset vertically by  $-0.2$  for clarity. The intralayer screening and ground-plane screening both contribute to the total screening, albeit on different length scales. This can be seen by comparing the data in Fig. 4(a) to that in Fig. 3(a), with the intralayer screening clearly the dominant contribution. As a result, distinguishing between individual traces in the sets corresponding to the metal (blue lines) or 2D (red lines) ground planes in Fig. 4(a) is difficult. Hence, to better quantify the enhancement that the ground-plane gives over intralayer screening alone, in Fig. 4(b) we plot the relative ground-plane enhancement  $R_{\text{metal,s}} = (\epsilon_{\text{single}}^{-1} - \epsilon_{\text{metal,s}}^{-1}) / |\epsilon_{\text{single}}^{-1}|$  (solid blue lines) and  $R_{\text{2D,s}} = (\epsilon_{\text{single}}^{-1} - \epsilon_{\text{2D,s}}^{-1}) / |\epsilon_{\text{single}}^{-1}|$  (dashed

red lines), respectively. Note that the ground-plane only provides significant enhancement over intralayer screening alone as  $d^{TF}$  becomes small, and as in Sec. II A, only provides enhancements at small  $q/q_{TF}$ . The small discrepancies between the data for the metal and 2D screening layers in Figs. 4(b) directly reflect the increased effectiveness of the metal ground-plane over a 2D ground-plane shown in Fig. 3(b).

The data in Figs. 3 and 4 provide an interesting insight into recent experiments on ground-plane screening in 2D hole systems in the insulating and metallic regimes.<sup>13,14</sup> Due to the low hole density and conductivity in the insulating regime, intralayer screening is less effective and the dominant contribution to screening is the ground-plane, which acts to limit the length scale of the Coulomb interactions, as Fig. 3(a) shows. This results in the ground-plane having a marked effect on the transport properties of the 2D system as shown by Huang *et al.*<sup>13</sup> In comparison, for the metallic state, where the density and conductivity are much higher, intralayer screening is the dominant contribution, and a ground-plane only acts as a long-range perturbation to the screening, as shown in Fig. 4(a). This perturbation to the intralayer screening is particularly small at  $d^{TF}=56.1$  and results in the ground-plane having relatively little effect on the transport properties in the metallic regime as found by Ho *et al.*<sup>14</sup> Although Fig. 4(b) suggests that decreasing  $d^{TF}$  will increase the effect of the ground plane, in practice there are issues in achieving this. For holes in GaAs, there is little scope for further reducing  $d$  due to increasing Coulomb drag and interlayer tunnelling effects. Also, in our model we have neglected interlayer exchange and correlation effects and these may become significant at these lower distances.

### C. Beyond the Thomas-Fermi approximation

Following our relatively simple treatment of ground-plane screening above, it is now interesting to ask how the results of our calculations change if we extend our model to account for two phenomena ignored in our Thomas-Fermi model: exchange effects at low densities and the finite thickness of the screening and transport layers.

The inclusion of the Hubbard local-field correction  $G(q)$ , finite thickness form factor  $F(q)$ , and the use of the Lindhard function for  $\chi^0(q)$  adds two parameters to the analysis, the well thickness  $a=20$  nm and the Fermi wave-vector  $k_F$ . This removes our ability to reduce the problem down to a single adjustable parameter  $d^{TF}$  as we did in Secs. III A and III B. Additionally, accounting for finite well width puts a lower limit on  $d$ , which must be greater than  $a$  to ensure that the wells remain separate. Hence for the remaining analysis we will only consider  $d=50$  and 30 nm, which correspond to  $d^{TF}=56.1$  and 33.7, respectively. As in earlier sections, we will first analyze the dielectric function ignoring intralayer screening in the transport layer, which we achieve by setting  $\rho_1^{ind}=0$ .

In Fig. 5(a), we plot  $\epsilon_{2D,ns,xf}^{-1}$  (dashed red lines) obtained using Eq. (20) for  $d=50$  and 30 nm, and for comparison,  $\epsilon_{metal,ns}^{-1}$  for  $d=50$  nm (dotted green line) from Fig. 3(a) and the corresponding result for  $d=30$  nm (solid blue line). One

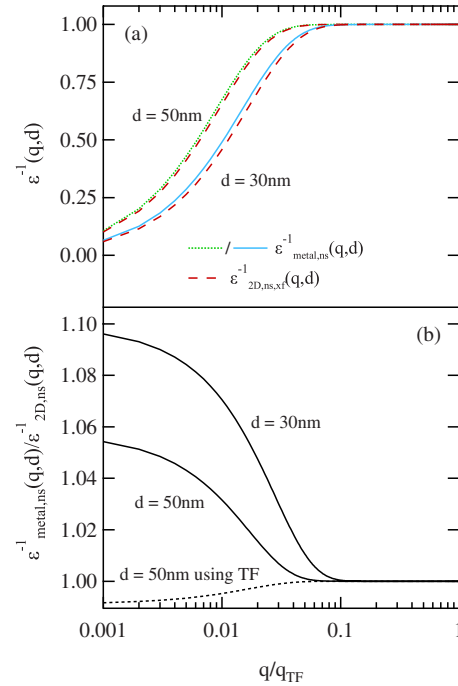


FIG. 5. (Color online) (a) Plots of  $\epsilon^{-1}$  vs  $q/q_{TF}$  for a metal (solid blue and dotted green line) and 2D (dashed red lines) ground plane for  $d=30$  and 50 nm accounting for exchange and finite thickness effects but ignoring intralayer screening in the transport layer. The dotted green line corresponds to that in Fig. 3(a). (b) A plot of the relative screening effect of a 2D layer compared to a metal (solid lines), as quantified by the ratio  $\epsilon_{metal,ns}^{-1}/\epsilon_{2D,ns}^{-1}$ . In contrast to the results for the Thomas-Fermi model [dashed line—data from Fig. 3(b)], we find that a 2D layer is actually more effective than a metal as a ground-plane when exchange and finite thickness effects are included in the calculation.

of the more significant effects of exchange (and also correlations) in 2D systems is that it leads to negative compressibility<sup>15,32,43</sup> for  $r_s \geq 2$ . It is well known from the field penetration experiments of Eisenstein *et al.*<sup>15</sup> that negative compressibility is related to an overscreening of the applied electric field, leading to a negative penetration field. In our calculations, a similar overscreening is observed, with a 2D system producing more effective ground-plane screening than a metal gate at intermediate  $q/q_{TF}$  in Fig. 5(a). The enhanced screening when the ground-plane is a 2D system is evident in Fig. 5(b), where we plot the ratio  $\epsilon_{metal,ns}^{-1}/\epsilon_{2D,ns}^{-1}$ , which takes values greater than 1 for  $q/q_{TF} \leq 0.1$ .

We now reintroduce intralayer screening in the transport layer, and in Fig. 6(a) we plot  $\epsilon_{metal,s,xf}^{-1}$  (solid blue line) and  $\epsilon_{2D,s,xf}^{-1}$  (dashed red line) for  $d=50$  and 30 nm. The values for the 2D system are offset vertically by  $-0.2$  for clarity. For comparison, we also plot  $\epsilon_{single}^{-1}$  (dash-dotted black lines—duplicated and offset vertically by  $-0.2$ ), along with the data from Fig. 3(a) for a metal ground-plane at  $d=50$  nm with no intralayer screening (green dashed line), and the expectation with no screening  $\epsilon^{-1}=1$  for all  $q/q_{TF}$  (grey dashed horizontal line). As we found earlier with the Thomas-Fermi model [see Fig. 4(a)], the inclusion of intralayer screening has a profound effect on the dielectric function contributing significantly more to the overall screening than the addition of a



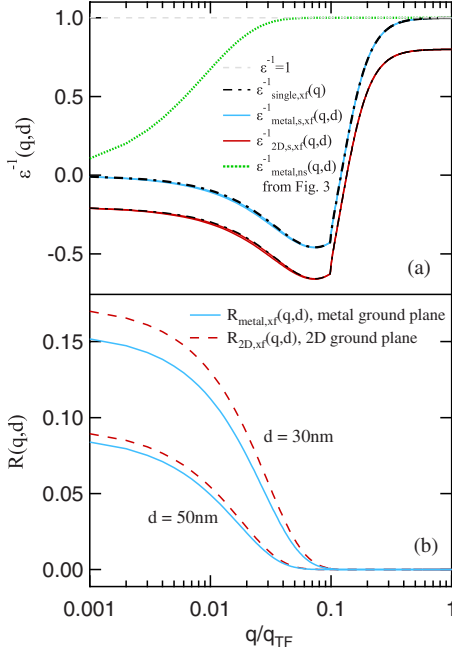


FIG. 6. (Color online) Effect of a screening layer on a 2D system with intralayer screening, with exchange and finite thickness effects included. (a) Firstly, in order to show the relative effects of intralayer and ground-plane screening, we plot the dielectric functions  $\epsilon^{-1}=1$  corresponding to no intralayer or ground-plane screening (grey dashed horizontal line),  $\epsilon_{metal,ns}^{-1}$  with metal screening layer at  $d=50$  nm and no intralayer screening [dotted green line, data from Fig. 3(a)], and  $\epsilon_{single,xf}^{-1}$  with intralayer screening but no ground-plane (dash-dotted black line). We then consider the effect of the metal screening layer when the intralayer screening is included, by plotting  $\epsilon_{metal,s,xf}^{-1}(q,d)$  (solid blue lines) for  $d=30$  and 50 nm. A similar set of curves are shown for the case of a 2D screening layer ( $\epsilon_{2D,s,xf}^{-1}(q,d)$ , solid red lines) which has been offset vertically by  $-0.2$  for clarity, along with a duplicate of  $\epsilon_{single,xf}^{-1}$  (dash-dotted black line). Since the dielectric functions almost lie on top of each other when intralayer screening is present, in (b) we plot  $R$ , the relative enhancement of  $\epsilon^{-1}$  due to the ground plane. Calculations for  $d=30$  and 50 nm are shown for metal (solid blue lines) and 2D (dashed red lines) ground-planes.

ground-plane does alone. This demonstrates the robustness of one of the key results of Sec. III B, namely, that in the metallic regime,<sup>14</sup> where intralayer screening effects are significant, the ground-plane screening contribution is overwhelmed by the intralayer screening contribution. This leads to a significantly reduced ground-plane effect than one would expect from studies in the insulating regime.<sup>13</sup>

The effect of including exchange and finite thickness effects in the calculation is evident by comparing  $\epsilon_{2D,s,xf}^{-1}$  in Fig. 6(a) with  $\epsilon_{2D,s}^{-1}$  in Fig. 4(a). Considering the individual contributions, because  $F(q) \leq 1$ , the finite thickness of the quantum well acts to reduce the effectiveness of the 2D layer as a ground-plane. In contrast, the negative compressibility produced by the exchange contribution acts to significantly enhance the screening, and as Fig. 6(a) shows, has its most significant impact at intermediate  $q/q_{TF}$ , where the dielectric function becomes negative, as discussed by Dolgov, Kirzhnits and Maksimov,<sup>33</sup> Ichimaru,<sup>34</sup> and Iwamoto.<sup>35</sup> The com-

bined effect of  $G(q)$  and  $F(q)$  is to significantly enhance the screening at intermediate  $q/q_{TF}$  whilst reducing it to levels comparable to the metal ground-plane for large  $q/q_{TF}$ . In other words, the added density dependence in our Hubbard model leads to enhanced midrange screening at the expense of short-range screening. A physical interpretation for this behavior is that at low densities there are insufficient carriers available to screen effectively close to a test charge, whilst at intermediate ranges, the negative compressibility produced by exchange leads to a higher availability of carriers and better screening than there would otherwise be at higher carrier densities where exchange is not as significant. It is also interesting to consider why the introduction of exchange and finite thickness effects have such a profound effect on the intralayer screening contribution compared to the ground-plane screening contribution. This occurs because the impact of  $G(q)$  and  $F(q)$  on the ground-plane contribution is strongly attenuated by the  $e^{-2qd}$  terms that appear in Eqs. (18) and (20). Such terms don't occur for the intralayer screening contribution, which significantly enhances the impact of the negative compressibility, as is clear by comparing Fig. 6(a) with Fig. 5(a).

We close by considering the relative effectiveness of the metal and 2D ground-planes with all considerations included in the calculations. In Fig. 6(b) we plot the relative ground-plane enhancements  $R_{metal,s,xf} = (\epsilon_{single,xf}^{-1} - \epsilon_{metal,s,xf}^{-1}) / |\epsilon_{single,xf}^{-1}|$  (solid blue lines) and  $R_{2D,s,xf} = (\epsilon_{single,xf}^{-1} - \epsilon_{2D,s,xf}^{-1}) / |\epsilon_{single,xf}^{-1}|$  (dashed red lines) for  $d=50$  and 30 nm. As in Fig. 5(b), we find that exchange, finite thickness and intralayer screening result in the 2D ground-plane screening *more* effectively than a metal ground-plane, with the difference between the two becoming greater as  $d$  is decreased. For  $d=50$  nm, the ground-plane separation used in Ref. 14, the ground-plane has significantly more effect ( $\sim 8-9\%$ ) than it does in the more simple Thomas-Fermi model ( $\sim 1\%$ ) presented earlier.

#### IV. SUMMARY OF RESULTS AND COMPARISON WITH EXPERIMENT

We have performed theoretical calculations to investigate the relative effectiveness of using a metal layer and a 2D system as a ground-plane to screen Coulomb interactions in an adjacent 2D system. This is done for two cases: the first is the relatively simple Thomas-Fermi approximation and the second is the Hubbard approximation, where we account for exchange and also finite thickness effects. This study was motivated by recent experiments of the effect of ground-plane screening on transport in semiconductor-based 2D systems.

There were three key findings to our study. Firstly, a 2D system is effective as a ground-plane for screening Coulomb interactions in a nearby 2D system, which was an open question following the recent experiment by Ho *et al.*<sup>14</sup> In the Thomas-Fermi approximation, a metal and a 2D system are almost equally effective at screening the long-range Coulomb interactions in the nearby 2D system, with the metal becoming relatively more effective as the ground-plane separation  $d$  is decreased.



Secondly, our calculations provide an explanation for why ground-plane screening has much more effect in the insulating regime than it did in the metallic regime. Due to the low hole density and conductivity in the insulating regime, intralayer screening is weak and the dominant contribution to screening is the ground-plane, which acts to limit the length scale of the Coulomb interactions. This results in the ground-plane having a marked effect on the transport properties of the 2D system, as shown by Huang *et al.*<sup>13</sup> In the metallic regime, intralayer screening cannot be ignored. In addition to being the dominant contribution for long-range interactions (i.e., at small  $q$ ), the intralayer screening contribution is non-zero over a much wider range of  $q$ , turning the ground-plane contribution into little more than a small change to the overall screening in the 2D system, which is consistent with the experiment by Ho *et al.*<sup>14</sup>

Finally, since both experiments were performed at  $r_s \gg 1$ , where the Thomas-Fermi approximation is invalid, we reconsider our calculations involving 2D systems using the Hubbard approximation for the local-field correction. We show that our argument regarding the physics of ground-plane screening in the metallic and insulating regimes remains robust, but that exchange effects lead to a 2D system being *more* effective than a metal layer as a ground-plane. This is due to the exchange-driven negative compressibility that occurs<sup>32</sup> at  $r_s \gtrsim 2$ .

## V. FURTHER WORK

While our results suggest that ground-plane effects on a metallic transport layer should strengthen as the ground-plane separation  $d^{TF}$  is reduced, there are a number of issues that complicate this argument. Firstly, for holes in GaAs, such as the experiment in Ref. 14, there is little scope to further reduce  $d$  due to the increasing Coulomb drag and interlayer tunnelling effects that would result. However, it may be possible to experimentally modify  $d^{TF}$  by moving to a different material system where  $(q^{TF})^{-1}$  is larger. For example, in InAs,<sup>36</sup> where  $m^* = 0.026m_e$  and  $\epsilon = 14.6\epsilon_0$ , we would have  $(q^{TF})^{-1} = 14.9$  nm, or InSb (Ref. 37) where  $m^* = 0.0145m_e$  and  $\epsilon = 17.7\epsilon_0$  gives  $(q^{TF})^{-1} = 32.3$  nm. These  $(q^{TF})^{-1}$  values are 17 and 28 times larger than those in Ref. 14, respectively. This would allow us to reduce  $d_{TF}$  without changing  $d$ , thus avoiding the problems above.

We note that our model neglects interlayer exchange and correlation effects, which may become significant at these small distances  $d$ , as suggested by calculations at  $r_s = 4$  by Liu *et al.*<sup>38</sup> It could be interesting to investigate the effect of including the interlayer exchange and correlation effects on the ground plane screening at small  $d$ . This may require using better approximations for the local field correction such as that developed by Singwi, Tosi, Land and Sjölander<sup>39</sup> (STLS), as there is no equivalent to the Hubbard approximation for interlayer local-field corrections.

We also note that using the technique in Ref. 14 and the theory presented here, it would be possible to study the breakdown of intralayer screening in the transport layer as it is evolved from the metallic to insulating regime. This could be compared with compressibility measurements of a 2D

system across the apparent metal-insulator transition,<sup>23</sup> possibly providing new insight into the mechanism driving this transition.

Lastly, in this paper we only calculate the screening of the ground-plane on the transport layer via the dielectric function. It would be interesting to take this work further to calculate the effect of the ground-plane on the actual carrier transport through the transport layer. Combining the theory presented here and various models of the metallic and insulating behaviors (see review papers),<sup>6–8</sup> it may be possible to determine how each of the models are affected by the presence of a ground plane, and would allow us compare this with the experimental data in more detail.

## ACKNOWLEDGMENTS

This work was funded by Australian Research Council (ARC). L.H.H. acknowledges financial support from the UNSW and the CSIRO. We thank M. Polini, I.S. Terekhov, and F. Green for helpful discussions.

## APPENDIX A: BRIEF REVIEW OF SCREENING THEORY FOR A SINGLE 2D SYSTEM

In this section we briefly review the basics of screening in a single 2D system. Readers familiar with screening theory may wish to proceed directly to Sec. II. A more extended discussion can be found in Refs. 21, 22, and 40.

Screening occurs when the carriers in a 2D system reorganize themselves in response to some added “external” positive charge density leading to an electrostatic potential determined by Poisson’s equation. This reorganization produces a negative “induced” charge density that acts to reduce or “screen” the electric field of the external charge. In proceeding, it is mathematically convenient to instead treat the problem in terms of wavevectors ( $q$  space) so that the (intralayer) Coulomb potential  $V(r) = \frac{1}{4\pi\epsilon r}$  becomes  $V(q) = \frac{1}{2\epsilon q}$ .<sup>21</sup>

There are two key parameters of interest in an analysis of screening. The first is the polarizability  $\chi(q)$ , which relates the induced (screening) charge density  $\rho^{\text{ind}}(\mathbf{q})$  to the external (unscreened) potential  $\phi^{\text{ext}}(\mathbf{q})$

$$\rho^{\text{ind}}(\mathbf{q}) = \chi(q)\phi^{\text{ext}}(\mathbf{q}). \quad (\text{A1})$$

The second is the dielectric function  $\epsilon(q)$ , which relates the total (screened) potential  $\phi(\mathbf{q})$  to the external (unscreened) potential  $\phi^{\text{ext}}(\mathbf{q})$

$$\phi(\mathbf{q}) = \phi^{\text{ext}}(\mathbf{q})/\epsilon(q). \quad (\text{A2})$$

Conceptually, the polarizability describes how much induced charge density is produced in response to the addition of the external charge density, hence it is also often called the density-density response function.<sup>41</sup> The dielectric function is a measure of how effective the screening is:  $\epsilon^{-1} = 1$  corresponds to no screening and  $\epsilon^{-1} = 0$  corresponds to perfect screening.<sup>33</sup> The two parameters can be linked via  $\phi^{\text{ext}}$  and the Coulomb potential  $V(q)$ , such that

$$\frac{1}{\epsilon(q)} = 1 + V(q)\chi(q). \quad (\text{A3})$$

The results above are precise aside from the assumption of linear response. However, continuing further requires calculation of  $\chi(q)$ . This cannot be achieved exactly, and requires the use of approximations. In the simplest instances, a combination of the Thomas-Fermi<sup>19,21</sup> (TF) and random phase approximations<sup>18</sup> (RPA) can be used. However, to properly account for exchange and/or correlation, particularly at lower carrier densities, more sophisticated approximations, such as those developed by Hubbard<sup>42</sup> or Singwi, Tosi, Land and Sjölander<sup>39</sup> (STLS) should be used. For a single 2D layer, this leads to a correction to the induced charge

$$\rho^{\text{ind}}(\mathbf{q}) = \chi^0(q)\{\phi^{\text{ext}}(\mathbf{q}) + V(q)\rho^{\text{ind}}(\mathbf{q})[1 - G(q)]\} \quad (\text{A4})$$

where  $G(q)$  is the local field factor, and  $\chi^0$  is the 2D Lindhard function.<sup>20</sup> This results in

$$\chi(q) = \frac{\chi^0(q)}{1 - V(q)\chi^0(q)[1 - G(q)]}. \quad (\text{A5})$$

The local-field factor can be calculated in numerous ways.<sup>22</sup> In this work, we use the Hubbard approximation,<sup>41,42</sup> which gives a local-field factor

$$G(q) = \frac{q}{2\sqrt{q^2 + k_F^2}} \quad (\text{A6})$$

where  $k_F = \sqrt{2\pi p}$  is the Fermi wavevector. Although better approximations are available,<sup>22</sup> the Hubbard approximation is sufficient to introduce a density-dependence into the

screening, unlike the Thomas-Fermi approximation, which is density independent.

## APPENDIX B: COMPARISON WITH OTHER WORK ON BILAYER SCREENING

In this Appendix, we discuss how the analytical expression we obtain for  $\epsilon_{2\text{D},s,\text{xf}}^{-1}(q, d)$  compares with other works on linear screening theory for bilayer 2D systems produced in double quantum well heterostructures, in particular, that of Zheng and MacDonald.<sup>30</sup> Note that we have translated the equations from Ref. 30 into the notation used in our paper for this Appendix.

Zheng and MacDonald begin by defining a density-density response function (polarizability)  $\chi_{ij}(q, \omega)$  for their bilayer 2D system by

$$\rho_i(q, \omega) = \sum_j \chi_{ij}(q, \omega) \phi_j^{\text{ext}}(q, \omega) \quad (\text{B1})$$

where  $\rho$  is the linear density response (i.e., induced-charge density),  $\phi_j^{\text{ext}}$  is the external potential and  $i, j=1, 2$  are the layer indices with 1 being the transport layer and 2 being the screening layer. Zheng and MacDonald then use the RPA<sup>18</sup> and STLS approximation<sup>39</sup> to obtain an expression for the polarizability:

$$\chi^{-1}(q, \omega) = \begin{pmatrix} [\chi_1^0(q, \omega)]^{-1} - V(q)[1 - G_{11}(q)] & U(q)[G_{12}(q) - 1] \\ U(q)[G_{21}(q) - 1] & [\chi_2^0(q, \omega)]^{-1} - V(q)[1 - G_{22}(q)] \end{pmatrix} \quad (\text{B2})$$

where  $G_{ij}(q)$  are the local-field factors that account for the effects of exchange and correlation.<sup>22</sup> For comparison with our work, we will consider  $\omega=0$  and ignore interlayer exchange and correlations by setting  $G_{12}(q)=G_{21}(q)=0$ ,  $G_{11}(q)=G_1(q)$ , and  $G_{22}(q)=G_2(q)$ . The latter approximation will be valid for large  $d$ , but we would expect that  $G_{ij}$  would become more significant at lower distances. This is seen in the work of Liu *et al.*,<sup>38</sup> in which  $G_{ii}$  and  $G_{ij}$  are calculated using STLS for different  $d$  at  $r_s=4$ .

In our work, we are seeking to obtain an effective single layer dielectric function for the transport layer only. Hence we only put external charge density  $\rho_1^{\text{ext}}(q)$  in the transport layer and set the external charge density in the screening layer  $\rho_2^{\text{ext}}(q)$  to zero. This results in external potentials in the two layers of  $\phi_1^{\text{ext}}(q)=V(q)\rho_1^{\text{ext}}(q)$ , and  $\phi_2^{\text{ext}}(q)=U(q)\rho_1^{\text{ext}}(q)$ . The total potential in the transport layer can thus be expressed as

$$\phi_1(q) = \phi_1^{\text{ext}}(q) + V(q)\rho_1^{\text{ind}}(q) + U(q)\rho_2^{\text{ind}}(q). \quad (\text{B3})$$

For the dielectric function of the transport layer, as defined in Eq. (3), this results in

$$\frac{1}{\epsilon(q, d)} = 1 + V(q)\chi_{11}(q) + U(q)\chi_{12}(q) + U(q)\chi_{21}(q) + e^{-qd}U(q)\chi_{22}(q). \quad (\text{B4})$$

This is analogous to Eq. (A3) for the single layer case. Indeed, by applying  $d \rightarrow \infty$  to Eq. (B4) reduces to Eq. (A3). Finally, obtaining the matrix elements  $\chi_{ij}(q)$  by inverting Eq. (B2) and inserting them into Eq. (B4), we obtain the same expression as that given for  $\epsilon_{2\text{D},s,\text{xf}}^{-1}(q)$  in Eq. (20) after returning to zero thickness [i.e.,  $F(q)=1$ ].

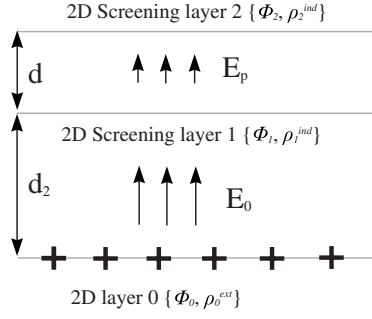


FIG. 7. Schematic showing the configuration used to calculate the penetration field across a 2D system.

### APPENDIX C: COMPARISON WITH THE SCREENING OF A PERPENDICULAR ELECTRIC FIELD BY A 2D SYSTEM

In this section we show that the calculations in this paper, which describe the screening of in-plane point charges (and also arbitrary in-plane charge distributions) by a 2D system and also an adjacent ground plane, are consistent with the equations by Eisenstein *et al.*<sup>15</sup> describing the penetration of a perpendicular electric field across a 2D system.

In order to calculate the penetration of the perpendicular electric field, it is necessary to consider a slightly different configuration than previously used in Fig. 2(b). Figure 7 shows a schematic of the system we now consider. We still have two 2D systems (labelled 1 and 2) separated by a distance  $d$ . We now have no external charge in either of these layers and only induced charges  $\rho_1^{\text{ind}}$  and  $\rho_2^{\text{ind}}$ . In order to apply an electric field across layer 1, we place a layer of external charge  $\rho_0^{\text{ext}}$  a distance  $d_2$  below layer 1. The net electric fields  $E_0$  and  $E_p$  are shown in Fig. 7, and have been related by Eisenstein *et al.* using Eqs. (5) and (6) in Ref. 15. Translating into the notation of our paper and using  $\chi_i^0 = -e^2(\frac{\partial n}{\partial \mu})_i$ , this results in

$$\frac{\partial E_p}{\partial E_0} = \frac{-\epsilon\chi_2^0}{-\epsilon(\chi_1^0 + \chi_2^0) + d\chi_1^0\chi_2^0}. \quad (\text{C1})$$

We now try to calculate the electric fields  $E_0$  and  $E_p$  directly using our model. The induced charges  $\rho_1^{\text{ind}}$  and  $\rho_2^{\text{ind}}$  are calculated in a similar fashion to previously in Sec. II C, although Eqs. (15) and (16) need to be modified slightly to take into account the position of the external charge. This results in

$$\rho_1^{\text{ind}}(\mathbf{q}) = \chi_1(\mathbf{q})[\phi_1^{\text{ext}}(\mathbf{q}) + U(\mathbf{q})\rho_2^{\text{ind}}(\mathbf{q})] \quad (\text{C2})$$

$$\rho_2^{\text{ind}}(\mathbf{q}) = \chi_2(\mathbf{q})[\phi_2^{\text{ext}}(\mathbf{q}) + U(\mathbf{q})\rho_1^{\text{ind}}(\mathbf{q})] \quad (\text{C3})$$

where

$$\phi_1^{\text{ext}}(\mathbf{q}) = V(\mathbf{q})e^{-q d_2}\rho_0^{\text{ext}}(\mathbf{q}) \quad (\text{C4})$$

$$\phi_2^{\text{ext}}(\mathbf{q}) = V(\mathbf{q})e^{-q(d+d_2)}\rho_0^{\text{ext}}(\mathbf{q}). \quad (\text{C5})$$

We set  $\rho_0^{\text{ext}}(\mathbf{r}) = \sigma_0$ , where  $\sigma_0$  is a constant 2D surface-charge density. This simplifies our model into the one-dimensional problem considered by Eisenstein *et al.*<sup>15</sup> In  $q$  space this gives  $\rho_0^{\text{ext}}(\mathbf{q}) = \sigma_0(2\pi)^2\delta(\mathbf{q})$ , where  $\delta(\mathbf{q})$  is the Dirac delta function. Solving Eqs. (C2) and (C3) simultaneously we obtain  $\rho_i^{\text{ind}}(\mathbf{q}) = \sigma_i(2\pi)^2\delta(\mathbf{q})$  with

$$\sigma_1 = \frac{\sigma_0\chi_1^0(\epsilon - d\chi_2^0)}{-\epsilon(\chi_1^0 + \chi_2^0) + d\chi_1^0\chi_2^0} \quad (\text{C6})$$

$$\sigma_2 = \frac{\sigma_0\epsilon\chi_2^0}{-\epsilon(\chi_1^0 + \chi_2^0) + d\chi_1^0\chi_2^0}, \quad (\text{C7})$$

where we have used the property of delta functions that  $f(\mathbf{q})\delta(\mathbf{q}) = f(0)\delta(\mathbf{q})$  for an arbitrary function  $f(\mathbf{q})$ . We note that the above Eqs. (C6) and (C7) are valid even for an arbitrary local field correction, as at  $q=0$  the Lindhardt function is equal to the Thomas-Fermi function and for any self-consistent local field correction  $G(0)=0$  (see equation 3.25 in Ref. 34). Similarly, the Eqs. (C6) and (C7) are valid even when allowing for the finite thickness of the 2D systems, since for an arbitrary form factor  $F(q)$  we have  $F(0)=1$ .

We can now calculate the electric fields  $E_0$  and  $E_p$ . For a single layer of uniform 2D charge density  $\sigma$ , the perpendicular electric field is given by  $E = \frac{\sigma}{2\epsilon}$ . By considering the positions of the electric field  $E_0$  and  $E_p$  with respect to the charge layers  $\rho_0$ ,  $\rho_1$ , and  $\rho_2$ , we obtain:

$$E_p = \frac{\sigma_0 + \sigma_1 - \sigma_2}{2\epsilon} \quad (\text{C8})$$

$$E_0 = \frac{\sigma_0 - \sigma_1 - \sigma_2}{2\epsilon}, \quad (\text{C9})$$

which results in

$$\frac{E_p}{E_0} = \frac{-\epsilon\chi_2^0}{-\epsilon(\chi_1^0 + \chi_2^0) + d\chi_1^0\chi_2^0}. \quad (\text{C10})$$

We note that in this model, the ratio  $\frac{E_p}{E_0}$  is equal to  $\frac{\partial E_p}{\partial E_0}$ , as we have neglected other charges commonly present in 2D systems, such as regions of modulation doping. Our resulting Eq. (C10) is thus in agreement with the equations used by Eisenstein *et al.*<sup>15</sup> (Eq. (C1)).

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<sup>1</sup>E. Wigner, Phys. Rev. **46**, 1002 (1934).

<sup>2</sup>R. S. Crandall and R. Williams, Phys. Lett. A **34**, 404 (1971).

<sup>3</sup>C. C. Grimes and G. Adams, Phys. Rev. Lett. **42**, 795 (1979).

<sup>4</sup>D. C. Tsui, H. L. Stormer, and A. C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982).

<sup>5</sup>R. B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983).

- <sup>6</sup>B. L. Altshuler, D. L. Maslov, and V. M. Pudalov, *Physica E* **9**, 209 (2001).
- <sup>7</sup>E. Abrahams, S. V. Kravchenko, and M. P. Sarachik, *Rev. Mod. Phys.* **73**, 251 (2001).
- <sup>8</sup>S. V. Kravchenko and M. P. Sarachik, *Rep. Prog. Phys.* **67**, 1 (2004).
- <sup>9</sup>F. M. Peeters, *Phys. Rev. B* **30**, 159 (1984).
- <sup>10</sup>A. Widom and R. Tao, *Phys. Rev. B* **38**, 10787 (1988).
- <sup>11</sup>H.-W. Jiang, M. A. Stan, and A. J. Dahm, *Surf. Sci.* **196**, 1 (1988).
- <sup>12</sup>G. Mistura, T. Günzler, S. Nesper, and P. Leiderer, *Phys. Rev. B* **56**, 8360 (1997).
- <sup>13</sup>J. Huang, D. S. Novikov, D. C. Tsui, L. N. Pfeiffer, and K. W. West, *arXiv:cond-mat/0610320*.
- <sup>14</sup>L. H. Ho, W. R. Clarke, A. P. Micolich, R. Danneau, O. Klochian, M. Y. Simmons, A. R. Hamilton, M. Pepper, and D. A. Ritchie, *Phys. Rev. B* **77**, 201402(R) (2008).
- <sup>15</sup>J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, *Phys. Rev. B* **50**, 1760 (1994).
- <sup>16</sup>J. D. Jackson, *Classical Electrodynamics*, 3rd ed. (John Wiley and sons, New York, 1999).
- <sup>17</sup>I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, 5th ed. (Academic Press, New York, 1993).
- <sup>18</sup>D. Bohm and D. Pines, *Phys. Rev.* **92**, 609 (1953).
- <sup>19</sup>L. H. Thomas, *Proc. Camb. Philos. Soc.* **23**, 542 (1927); E. Fermi, *Z. Phys.* **48**, 73 (1928).
- <sup>20</sup>F. Stern, *Phys. Rev. Lett.* **18**, 546 (1967).
- <sup>21</sup>J. H. Davies, *The Physics of Low-Dimensional Systems* (Cambridge University Press, Cambridge, 1998).
- <sup>22</sup>G. F. Giuliani and G. Vignale, *Quantum Theory of the Electron Liquid* (Cambridge University Press, Cambridge, 2005).
- <sup>23</sup>G. Allison, E. A. Galaktionov, A. K. Savchenko, S. S. Safonov, M. M. Fogler, M. Y. Simmons, and D. A. Ritchie, *Phys. Rev. Lett.* **96**, 216407 (2006).
- <sup>24</sup>J. Shi and X. C. Xie, *Phys. Rev. Lett.* **88**, 086401 (2002).
- <sup>25</sup>M. M. Fogler, *Phys. Rev. B* **69**, 121409(R) (2004).
- <sup>26</sup>T. Ando, A. B. Fowler, and F. Stern, *Rev. Mod. Phys.* **54**, 437 (1982).
- <sup>27</sup>F. Stern, *Jpn. J. Appl. Phys., Suppl.* **2**, 323 (1974).
- <sup>28</sup>A. Gold, *Phys. Rev. B* **35**, 723 (1987).
- <sup>29</sup>P. J. Price, *Phys. Rev. B* **30**, 2234 (1984).
- <sup>30</sup>L. Zheng and A. H. MacDonald, *Phys. Rev. B* **49**, 5522 (1994).
- <sup>31</sup>L. A. Tracy, E. H. Hwang, K. Eng, G. A. Ten Eyck, E. P. Nordberg, K. Childs, M. S. Carroll, M. P. Lilly, and S. Das Sarma, *Phys. Rev. B* **79**, 235307 (2009).
- <sup>32</sup>B. Tanatar and D. M. Ceperley, *Phys. Rev. B* **39**, 5005 (1989).
- <sup>33</sup>O. V. Dolgov, D. A. Kirzhnits, and E. G. Maksimov, *Rev. Mod. Phys.* **53**, 81 (1981).
- <sup>34</sup>S. Ichimaru, *Rev. Mod. Phys.* **54**, 1017 (1982).
- <sup>35</sup>N. Iwamoto, *Phys. Rev. B* **43**, 2174 (1991).
- <sup>36</sup>S. Adachi, *J. Appl. Phys.* **53**, 8775 (1982).
- <sup>37</sup>K. J. Goldammer, S. J. Chung, W. K. Liu, M. B. Santos, J. L. Hicks, S. Raymond, and S. Q. Murphy, *J. Cryst. Growth* **201-202**, 753 (1999).
- <sup>38</sup>L. Liu, L. Swierkowski, D. Neilson, and J. Szymański, *Phys. Rev. B* **53**, 7923 (1996).
- <sup>39</sup>K. S. Singwi, M. P. Tosi, R. H. Land, and A. Sjölander, *Phys. Rev.* **176**, 589 (1968).
- <sup>40</sup>N. W. Ashcroft and N. D. Mermin, *Solid State Physics* (Saunders College Publishing, Orlando, 1976).
- <sup>41</sup>M. Jonson, *J. Phys. C* **9**, 3055 (1976).
- <sup>42</sup>J. Hubbard, *Proc. R. Soc. Lond. A Math. Phys. Sci.* **243**, 336 (1958).
- <sup>43</sup>M. S. Bello, E. I. Levin, B. I. Shklovskii, and A. L. Efros, *Sov. Phys. JETP* **53**, 822 (1981).